Contents

Chapter 1

Matrix Calculus

A few basic rules for matrix calculus are given below. Denote:

$\mathbf{A},\ \mathbf{B}, \mathbf{C}$	matrices
$\mathbf{A}^T$	transpose of matrix $\mathbf{A}$
$\mathbf{A}^{-1}$	inverse of matrix $\mathbf{A}$
$\mathbf{b}$	vector
$\mathbf{x}$	a vector $[x_1 \ \ x_2 \ \ \ldots x_n]^T$
$\mathbf{y}$	a vector $[y_1 \ \ y_2 \ \ \ldots y_m]^T$

1.1 Transpose and inverse

$$(\mathbf{A}+\mathbf{B})^T = \mathbf{A}^T + \mathbf{B}^T \tag{1.1}$$

$$(\mathbf{AB})^T = \mathbf{B}^T\mathbf{A}^T \tag{1.2}$$

$$(\mathbf{ABC}\ldots)^T = \ldots\mathbf{C}^T\mathbf{B}^T\mathbf{A}^T \tag{1.3}$$

$$(\mathbf{AB})^{-1} = \mathbf{A}^{-1}\mathbf{B}^{-1} \tag{1.4}$$

$$(\mathbf{ABC}\ldots)^{-1} = \ldots\mathbf{C}^{-1}\mathbf{B}^{-1}\mathbf{A}^{-1} \tag{1.5}$$

$$(\mathbf{A}^T)^{-1} = (\mathbf{A}^{-1})^T \tag{1.6}$$

1.2 Matrix differentiation

$$\frac{\partial}{\partial \mathbf{x}}(\mathbf{Ax}) = \mathbf{A}^T \tag{1.7}$$

$$\frac{\partial}{\partial \mathbf{x}}\left(\mathbf{x}^T \mathbf{A}\right) = \mathbf{A} \tag{1.8}$$

$$\frac{\partial}{\partial \mathbf{x}}\left(\mathbf{x}^T \mathbf{A}\mathbf{x}\right) = \left(\mathbf{A}^T + \mathbf{A}\right)\mathbf{x} \tag{1.9}$$

If $\mathbf{A}$ is symmetric, $\mathbf{A} = \mathbf{A}^T$, and:

$$\frac{\partial}{\partial \mathbf{x}}\left(\mathbf{x}^T \mathbf{A}\mathbf{x}\right) = 2\mathbf{A}\mathbf{x} \tag{1.10}$$

$$\frac{\partial}{\partial \mathbf{x}}\left(\mathbf{x}^T \mathbf{x}\right) = 2\mathbf{x} \tag{1.11}$$

$$\frac{\partial}{\partial \mathbf{x}}\left(\mathbf{x}^T \mathbf{x}\right) = 2\mathbf{x} \tag{1.12}$$

If $\mathbf{x}$ and $\mathbf{y}$ are $n \times 1$ and $m \times 1$ constant vectors and $\mathbf{A}$ is an $n \times m$ matrix:

$$\frac{\partial}{\partial \mathbf{A}}\left(\mathbf{x}^T \mathbf{A}\mathbf{y}\right) = \mathbf{x}\mathbf{y}^T \tag{1.13}$$

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